Prof. D. Chaudhury

Particle velocity,

$$\dot{u} = \frac{\partial u}{\partial t} = \frac{\epsilon_x . \partial x}{\partial t}$$

$$\boxed{\because\ \epsilon_x = \frac{\partial u}{\partial x}} = \frac{\sigma_x}{M} \cdot \frac{v_p . \partial t}{\partial t}$$

$$\boxed{\because\ \sigma_x = M . \epsilon_x} = \frac{\sigma_x}{M} . v_p$$

$$\dot{u} = \frac{\partial u}{\partial t} = \frac{\sigma_x}{M} \cdot v_p$$

$$= \frac{\sigma_x}{\rho v_p^{\cancel{2}}} \cdot \cancel{v_p}$$

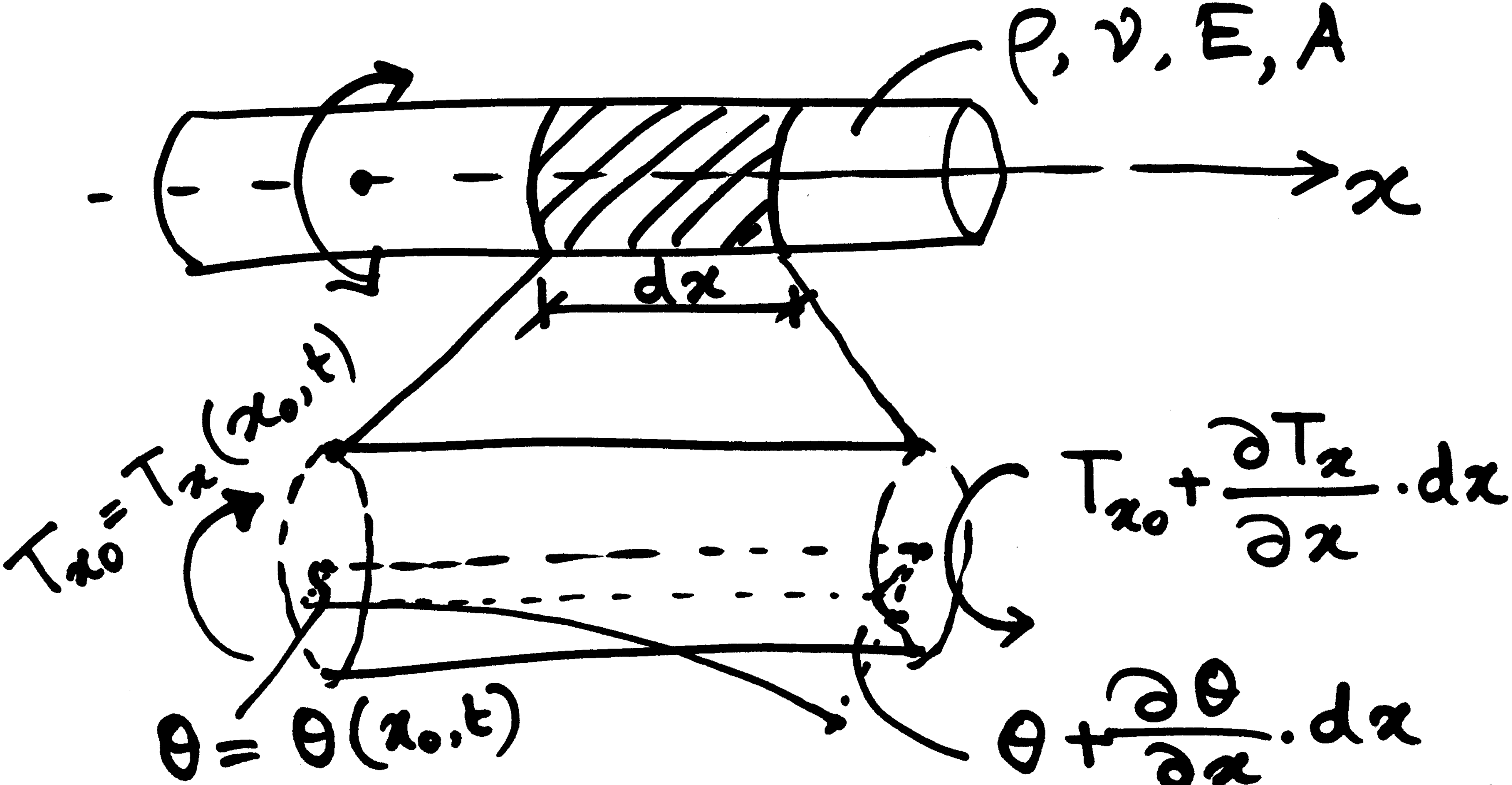
$$\left[\because \frac{M}{\rho} = v_p^2 \right] = \frac{\sigma_x}{\rho v_p}$$

$$\boxed{\dot{u} = \frac{\sigma_x}{\rho v_p}}$$

$\rho v_p \rightarrow$ specific impedance

DC/2

Torsional Wave in Infinite Rod



$$-\cancel{T_{x_0}}+\left[\cancel{T_{x_0}}+\frac{\partial T_x}{\partial x}\cdot d\cancel{x}\right]$$

$$=\left(\rho\cdot J\cdot d\cancel{x}\right)\cdot\frac{\partial^2\theta}{\partial t^2}$$

$$[\because dx\neq 0]$$

$$\boxed{\frac{\partial T_x}{\partial x}=\rho J\cdot\frac{\partial^2\theta}{\partial t^2}}$$

Torque

Rotation

$$T_x = \underset{\uparrow}{G}J \cdot \frac{\partial \theta}{\partial x} \Rightarrow \frac{\partial T_x}{\partial x} = GJ \cdot \frac{\partial^2 \theta}{\partial x^2}$$

shear modulus

$$\frac{\partial^2 \theta}{\partial t^2} = \frac{1}{\rho J} \cdot \frac{\partial T_x}{\partial x}$$

or, $$\frac{\partial^2 \theta}{\partial t^2} = \frac{1}{\rho \cancel{J}} \cdot G\cancel{J} \cdot \frac{\partial^2 \theta}{\partial x^2} \quad (\because J \neq 0)$$

$$\frac{\partial^2\theta}{\partial t^2} = \left(\frac{G}{\rho}\right)\frac{\partial^2\theta}{\partial x^2}$$

$\frac{G}{\rho} = v_s^2$, where, $v_s \rightarrow$ shear wave velocity.

$$v_s = \sqrt{\frac{G}{\rho}}$$

$$\therefore \boxed{\frac{\partial^2\theta}{\partial t^2} = v_s^2 \cdot \frac{\partial^2\theta}{\partial x^2}}$$

General form of equation of motion in 1-D.

$$\boxed{\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2}}$$

Loading cond^{n}s

General form of solution,

$$u(x,t) = f(vt - x) + g(vt + x)$$

wave is travelling in $+x$ dir.

wave is travelling in $-x$ dir.

DC/7

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Lec 18(B), 28/2/13

Harmonic loading

$$\sigma(t) = \sigma_0 \cos(\bar{\omega} t)$$

Harmonic Response,

$$u(x,t) = A\cos(\bar{\omega}t - kx) + B\cos(\bar{\omega}t + kx)$$

$$K = \frac{\bar{\omega}}{v} \text{ (wave number)}$$

$$\lambda = vT = \frac{v}{f} = \frac{2\pi}{\omega} \cdot v$$

$$= \frac{2\pi}{k}$$

wavelength

wave number

$$\lambda = \frac{2\pi}{k} \rightarrow \text{'}x\text{' co-ordinate system}$$

$$T = \frac{2\pi}{\omega} \rightarrow \text{'}t\text{' co-ordinate system}$$

DC/2